

# Lecture 3: Integer representation

David Hovemeyer

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# Integer representation

# Representing integers

- ▶ We've seen how to represent unsigned (nonnegative) integers
  - ▶ Bit string interpreted as a binary (base 2) number
- ▶ How to represent signed integers?
  - ▶ Sign magnitude
  - ▶ Ones' complement
  - ▶ Two's complement
- ▶ In examples that follow, we'll use 4-bit words
  - ▶ Ideas will generalize to larger word sizes

# Desired features for signed representation

What we want in a representation for signed integers:

- ▶ About half of encoding space used for negative values
- ▶ Each represented integer has a unique encoding as bit string
- ▶ Straightforward way to do arithmetic

# Sign magnitude representation

Let most significant bit be a sign bit: **0**→positive, **1**→negative

Bit string	value	Bit string	value
<b>0000</b>	0	<b>1000</b>	-0
<b>0001</b>	1	<b>1001</b>	-1
<b>0010</b>	2	<b>1010</b>	-2
<b>0011</b>	3	<b>1011</b>	-3
<b>0100</b>	4	<b>1100</b>	-4
<b>0101</b>	5	<b>1101</b>	-5
<b>0110</b>	6	<b>1110</b>	-6
<b>0111</b>	7	<b>1111</b>	-7

Downsides: two representations of 0, arithmetic complicated by sign bit

# Ones' complement

Ones' complement: to represent  $-x$ , invert all of the bits of  $x$

Bit string	value	Bit string	value
0000	0	1000	-7
0001	1	1001	-6
0010	2	1010	-5
0011	3	1011	-4
0100	4	1100	-3
0101	5	1101	-2
0110	6	1110	-1
0111	7	1111	-0

Downsides: two representations of 0, slightly complicated arithmetic

# Sign magnitude and ones' complement are obsolete

- ▶ Sign magnitude and ones' complement representations are not used for integer representation by modern computers
  - ▶ But, sign magnitude is used in floating point representation
- ▶ The rest of this lecture will discuss *two's complement*

# Two's complement

Two's complement: in  $w$ -bit word, the most significant bit represents  $-2^{w-1}$

E.g., when  $w = 4$ ,

Representation	Bit 3	Bit 2	Bit 1	Bit 0
Unsigned	8	4	2	1
Two's complement	-8	4	2	1

Given bit string 1011,

- ▶ Unsigned, 1011 is  $8 + 2 + 1 = 11$
- ▶ Two's complement, 1011 is  $-8 + 2 + 1 = -5$



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0001	1	1001	-7
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0011	3	1011	-5
0100	4	1100	-4
0101	5	1101	-3
0110	6	1110	-2
0111	7	1111	-1

Note asymmetry of negative and positive ranges: -8 is represented, 8 isn't

# Thinking about two's complement

Useful way to think about a  $w$ -bit two's complement representation:

- ▶ Bit  $w - 1$  is the sign bit, 0→positive, 1→negative
- ▶ If sign bit is 0, usual unsigned interpretation
- ▶ If sign bit is 1, bits  $w - 2 \dots 0$  indicate the “offset” from  $-2^{w-1}$

# Two's complement example

Given  $w = 4$ , example bit string is 1011

- ▶ Sign bit is 1
- ▶ Offset from  $-2^3$  is 011, which is 3 ( $2+1$ )
- ▶  $-8 + 3 = -5$

So, 1011 represents -5

Clicker quiz omitted from public slides

# Why two's complement?

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Wow!

# Trying it out

Add two 8 bit integer values:

00101101



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# Trying it out

As unsigned values:

$$\begin{array}{r} 00101101 \quad 45 \\ + 11111100 \quad 252 \\ \hline 100101001 \quad 297 \end{array} \quad (\text{truncated to } 41)$$

# Trying it out

As signed two's complement values:

$$\begin{array}{r} 00101101 \quad 45 \\ + \quad 11111100 \quad -4 \\ \hline 100101001 \quad 41 \end{array}$$

# Subtraction via addition

- ▶ Two's complement negation: invert all bits, then add 1
- ▶ Example, negating 5
  - ▶ Original value: 00000101
  - ▶ Invert bits: 11111010
  - ▶ Add one: 11111011
  - ▶ Value is  $-128 + 64 + 32 + 16 + 8 + 2 + 1 = -5$
- ▶  $a - b$  can be computed as  $a + -b$ 
  - ▶ I.e., invert  $b$ , then add to  $a$

# Sign extension

- ▶ Sometimes it is necessary to increase the number of bits in the representation of a signed integer
  - ▶ E.g., type cast or implicit conversion of a 16 bit `short` value to a 32 bit `int` value
- ▶ In two's complement, this can be accomplished by *sign extension*: replicate the original sign bit as many times as necessary
  - ▶ This preserves the numeric value!
  - ▶ Processors typically have dedicated instructions to perform sign extension

# Sign extension example

Example: extend 4 bit two's complement values 1011 and 0011 to 8 bits

Number of bits	Bit string	Meaning
4	<u>1</u> 011	$-8 + 2 + 1 = -5$
8	<b>1111</b> <u>1</u> 011	$-128 + 64 + 32 + 16 + 8 + 2 + 1 = -5$
4	0 <u>0</u> 11	$2 + 1 = 3$
8	<b>0000</b> <u>0</u> 011	$2 + 1 = 3$

# Sign extension example program

```
#include <stdio.h>

void printbits(int x, int n) {
    for (int i = n-1; i >= 0; i--) {
        putchar(x & (1 << i) ? '1' : '0');
    }
    putchar('\n');
}

int main(void) {
    short s = -27987;
    int i = (int) s;           // <-- sign extension occurs here
    printf("%*c", 16, ' ');
    printbits(s, 16);
    printbits(i, 32);
    return 0;
}
```



# Sign extension example program (output)

```
$ gcc signext.c
```

```
$ ./a.out
```

```
1001001010101101
```

```
11111111111111111001001010101101
```

# Clicker quiz!

Clicker quiz omitted from public slides

# Extending unsigned values

Extending the representation of an unsigned value is straightforward:  
unconditionally pad with 0 bits

Example: 4 bit unsigned value  $1011 = 8 + 2 + 1 = 11$

As an 8 bit unsigned value, **0000** $1011 = 8 + 2 + 1 = 11$

# General observation

In general, increasing the number of bits in the representation of an integer (signed or unsigned) will preserve its value

# Truncation

- ▶ Truncation: *reducing* the number of bits in the representation of an integer
  - ▶ In general, this will lose information and potentially change the value
- ▶ Truncation is done by chopping off bits from the left side of the bit string
  - ▶ Whatever remains is the new representation

# Truncation example

Example: convert signed 8 bit integer -14 to a 4 bit signed integer

Number of bits	Bit string	Meaning
8	11110010	$-128 + 64 + 32 + 16 + 2 = -14$
4	0010	2

# Truncation example program

```
#include <stdio.h>
void printbits(int x, int n) {
    for (int i = n-1; i >= 0; i--) {
        putchar(x & (1 << i) ? '1' : '0');
    }
    putchar('\n');
}

int main(void) {
    short s = -129;
    char c = s;      // <-- truncation occurs here
    printf("s=%d, c=%d\n", s, c);
    printbits(s, 16);
    printf("%*c", 8, ' ');
    printbits(c, 8);
    return 0;
}
```

# Truncation example program (output)

```
$ gcc truncate.c
$ ./a.out
s=-129, c=127
1111111101111111
      01111111
```

Explanation:

- ▶ short is a 16 bit signed type, char<sup>1</sup> is a signed 8 bit type
- ▶ After truncation from 16 to 8 bits, the sign bit was 0, so the resulting value became positive
- ▶ Look at the bit representations — convince yourself the values output by printf make sense!

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<sup>1</sup>Compiler-dependent, tested with gcc 7.4.0 on x86-64 Linux



# Conversions between signed and unsigned

- ▶ Another important type of conversion is between signed and unsigned values
- ▶ Fundamentally, data in the computer's memory has *no inherent meaning*
- ▶ It is up to the *program* to decide how to interpret data
- ▶ Conversions between signed and unsigned (without changing the number of bits) *do not change the underlying representation as bits*

# Signed/unsigned conversion examples

Example: bit pattern 10010110 as signed and unsigned 8 bit integer values

Signed:  $-128 + 16 + 4 + 2 = -106$

Unsigned:  $128 + 16 + 4 + 2 = 150$

# Signed/unsigned conversion example program

```
#include <stdio.h>
unsigned char parsebits(const char *s) {
    unsigned char val = 0;
    char c;
    while ((c = *s++)) {
        val <<= 1;
        if (c == '1') { val |= 1; }
    }
    return val;
}

int main(void) {
    unsigned char uc = parsebits("10010110");
    char c = (char) uc;    // <-- conversion from unsigned to signed
    printf("%u %d\n", uc, c);
    return 0;
}
```

# Signed/unsigned conversion example program (output)

```
$ gcc convert.c  
$ ./a.out  
150 -106
```

# Considerations for writing programs

# Programming considerations

- ▶ Semantics of integer values and data types can be surprisingly subtle
- ▶ C and C++ further complicate matters in several ways:
  - ▶ Data type sizes vary
  - ▶ Integer representation not actually specified by the language!
  - ▶ Some operations the program could perform have semantics that are implementation-defined or (worse) *undefined*
- ▶ Recommendation: be very careful!

# Implicit conversions

- ▶ In C, there are many contexts in which *implicit conversions* will occur
  - ▶ Including ones where information can be lost!
- ▶ It's important to know where implicit conversions happen and to understand their effects
- ▶ It's not a bad idea to use explicit type casts so that conversions are *explicit*, even if they aren't strictly necessary
  - ▶ Semantics of program are more obvious, avoid unintended behaviors

# Sign extension

- ▶ Sign extension can sometimes have surprising consequences (bits that you thought would be 0 become 1)
- ▶ Values belonging to unsigned types (unsigned char, unsigned short, etc.) are never sign extended