



Lecture 3: Integer representation

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Integer representation

Representing integers

- We've seen how to represent unsigned (nonnegative) integers
 - Bit string interpreted as a binary (base 2) number
- How to represent signed integers?
 - Sign magnitude
 - Ones' complement
 - Two's complement
- In examples that follow, we'll use 4-bit words
 - Ideas will generalize to larger word sizes



Desired features for signed representation

What we want in a representation for signed integers:

- About half of encoding space used for negative values
- Each represented integer has a unique encoding as bit string
- Straightforward way to do arithmetic



Sign magnitude representation

Let most significant bit be a sign bit: **0**→positive, **1**→negative

Bit string	value	Bit string	value
0 000	0	1 000	-0
0 001	1	1 001	-1
0 010	2	1 010	-2
0 011	3	1 011	-3
0 100	4	1 100	-4
0 101	5	1 101	-5
0 110	6	1 110	-6
0 111	7	1 111	-7

Downsides: two representations of 0, arithmetic complicated by sign bit

Ones' complement

Ones' complement: to represent $-x$, invert all of the bits of x

Bit string	value	Bit string	value
0000	0	1000	-7
0001	1	1001	-6
0010	2	1010	-5
0011	3	1011	-4
0100	4	1100	-3
0101	5	1101	-2
0110	6	1110	-1
0111	7	1111	-0

Downsides: two representations of 0, slightly complicated arithmetic



Sign magnitude and ones' complement are obsolete

- Sign magnitude and ones' complement representations are not used for integer representation by modern computers
 - But, sign magnitude is used in floating point representation
- The rest of this lecture will discuss *two's complement*



Two's complement

Two's complement: in w -bit word, the most significant bit represents -2^{w-1}

E.g., when $w = 4$,

Representation	Bit 3	Bit 2	Bit 1	Bit 0
Unsigned	8	4	2	1
Two's complement	-8	4	2	1

Given bit string 1011,

- Unsigned, 1011 is $8 + 2 + 1 = 11$
- Two's complement, 1011 is $-8 + 2 + 1 = -5$

Two's complement

Two's complement: in w -bit word, the most significant bit represents -2^{w-1}

Bit string	value	Bit string	value
0000	0	1000	-8
0001	1	1001	-7
0010	2	1010	-6
0011	3	1011	-5
0100	4	1100	-4
0101	5	1101	-3
0110	6	1110	-2
0111	7	1111	-1

Note asymmetry of negative and positive ranges: -8 is represented, 8 isn't



Thinking about two's complement

Useful way to think about a w -bit two's complement representation:

- Bit $w - 1$ is the sign bit, $0 \rightarrow$ positive, $1 \rightarrow$ negative
- If sign bit is 0, usual unsigned interpretation
- If sign bit is 1, bits $w - 2 \dots 0$ indicate the “offset” from -2^{w-1}



Two's complement example

Given $w = 4$, example bit string is 1011

- Sign bit is 1
- Offset from -2^3 is 011, which is 3 ($2+1$)
- $-8 + 3 = -5$

So, 1011 represents -5



Clicker quiz

Clicker quiz omitted from public slides



Why two's complement?

The most important advantage of two's complement:



Why two's complement?

The most important advantage of two's complement:

Unsigned addition yields correct
result for signed values!



Why two's complement?

The most important advantage of two's complement:

Unsigned addition yields correct
result for signed values!

Wow!



Trying it out

Add two 8 bit integer values:

00101101



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$$\begin{array}{r} 00101101 \\ + 11111100 \\ \hline \end{array}$$



Trying it out

Add two 8 bit integer values:

$$\begin{array}{r} 00101101 \\ + 11111100 \\ \hline 100101001 \end{array}$$



Trying it out

As unsigned values:

$$\begin{array}{r} 00101101 \quad 45 \\ + \quad 11111100 \quad 252 \\ \hline 100101001 \quad 297 \end{array} \quad (\text{truncated to } 41)$$



Trying it out

As signed two's complement values:

$$\begin{array}{r} 00101101 \quad 45 \\ + \quad 11111100 \quad -4 \\ \hline 100101001 \quad 41 \end{array}$$



Subtraction via addition

- Two's complement negation: invert all bits, then add 1
- Example, negating 5
 - Original value: 00000101
 - Invert bits: 11111010
 - Add one: 11111011
 - Value is $-128 + 64 + 32 + 16 + 8 + 2 + 1 = -5$
- $a - b$ can be computed as $a + -b$
 - I.e., invert b , then add to a



Sign extension

- Sometimes it is necessary to increase the number of bits in the representation of a signed integer
 - E.g., type cast or implicit conversion of a 16 bit `short` value to a 32 bit `int` value
- In two's complement, this can be accomplished by *sign extension*: replicate the original sign bit as many times as necessary
 - This preserves the numeric value!
 - Processors typically have dedicated instructions to perform sign extension



Sign extension example

Example: extend 4 bit two's complement values 1011 and 0011 to 8 bits

Number of bits	Bit string	Meaning
4	<u>1</u> 011	$-8 + 2 + 1 = -5$
8	1111 <u>1</u> 011	$-128 + 64 + 32 + 16 + 8 + 2 + 1 = -5$
4	<u>0</u> 011	$2 + 1 = 3$
8	0000 <u>0</u> 011	$2 + 1 = 3$



Sign extension example program

```
#include <stdio.h>

void printbits(int x, int n) {
    for (int i = n-1; i >= 0; i--) {
        putchar(x & (1 << i) ? '1' : '0');
    }
    putchar('\n');
}

int main(void) {
    short s = -27987;
    int i = (int) s;           // <-- sign extension occurs here
    printf("%*c", 16, ' ');
    printbits(s, 16);
    printbits(i, 32);
    return 0;
}
```



Sign extension example program (output)

```
$ gcc signext.c
$ ./a.out
                1001001010101101
111111111111111111001001010101101
```



Clicker quiz!

Clicker quiz omitted from public slides



Extending unsigned values

Extending the representation of an unsigned value is straightforward:
unconditionally pad with 0 bits

Example: 4 bit unsigned value $1011 = 8 + 2 + 1 = 11$

As an 8 bit unsigned value, **0000** $1011 = 8 + 2 + 1 = 11$



General observation

In general, increasing the number of bits in the representation of an integer (signed or unsigned) will preserve its value



Truncation

- Truncation: *reducing* the number of bits in the representation of an integer
 - In general, this will lose information and potentially change the value
- Truncation is done by chopping off bits from the left side of the bit string
 - Whatever remains is the new representation



Truncation example

Example: convert signed 8 bit integer -14 to a 4 bit signed integer

Number of bits	Bit string	Meaning
8	11110010	$-128 + 64 + 32 + 16 + 2 = -14$
4	0010	2



Truncation example program

```
#include <stdio.h>

void printbits(int x, int n) {
    for (int i = n-1; i >= 0; i--) {
        putchar(x & (1 << i) ? '1' : '0');
    }
    putchar('\n');
}

int main(void) {
    short s = -129;
    char c = s;      // <-- truncation occurs here
    printf("s=%d, c=%d\n", s, c);
    printbits(s, 16);
    printf("%*c", 8, ' ');
    printbits(c, 8);
    return 0;
}
```



Truncation example program (output)

```
$ gcc truncate.c
$ ./a.out
s=-129, c=127
1111111110111111
      01111111
```

Explanation:

- short is a 16 bit signed type, char is a signed 8 bit type (depends on compiler/system)
- After truncation from 16 to 8 bits, the sign bit was 0, so the resulting value became positive
- Look at the bit representations — convince yourself the values output by printf make sense!



Conversions between signed and unsigned

- Another important type of conversion is between signed and unsigned values
- Fundamentally, data in the computer's memory has *no inherent meaning*
- It is up to the *program* to decide how to interpret data
- Conversions between signed and unsigned (without changing the number of bits) *do not change the underlying representation as bits*



Signed/unsigned conversion examples

Example: bit pattern 10010110 as signed and unsigned 8 bit integer values

Signed: $-128 + 16 + 4 + 2 = -106$

Unsigned: $128 + 16 + 4 + 2 = 150$



Signed/unsigned conversion example program

```
#include <stdio.h>

unsigned char parsebits(const char *s) {
    unsigned char val = 0;
    char c;
    while ((c = *s++)) {
        val <<= 1;
        if (c == '1') { val |= 1; }
    }
    return val;
}

int main(void) {
    unsigned char uc = parsebits("10010110");
    char c = (char) uc;      // <-- conversion from unsigned to signed
    printf("%u %d\n", uc, c);
    return 0;
}
```



Signed/unsigned conversion example program (output)

```
$ gcc convert.c  
$ ./a.out  
150 -106
```



Considerations for writing programs

Programming considerations

- Semantics of integer values and data types can be surprisingly subtle
- C and C++ further complicate matters in several ways:
 - Data type sizes vary
 - Integer representation not actually specified by the language!
 - Some operations the program could perform have semantics that are implementation-defined or (worse) *undefined*
- Recommendation: be very careful!



Implicit conversions

- In C, there are many contexts in which *implicit conversions* will occur
 - Including ones where information can be lost!
- It's important to know where implicit conversions happen and to understand their effects
- It's not a bad idea to use explicit type casts so that conversions are *explicit*, even if they aren't strictly necessary
 - Semantics of program are more obvious, avoid unintended behaviors



Sign extension

- Sign extension can sometimes have surprising consequences (bits that you thought would be 0 become 1)
- Values belonging to unsigned types (`unsigned char`, `unsigned short`, etc.) are never sign extended



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